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Multi-conjugate adaptive optics with hybrid laser beacon systems

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ABSTRACT

Results for a multi-conjugate adaptive optical (MCAO) system with multiple laser beacons at multiple altitudes are presented in this paper. The use of multi-conjugate deformable mirrors (DM's) increases the corrected field of view of an adaptive optical telescope system. This improves the imaging capability for extended astronomical objects such as planets, galaxies, and nebulae. Multiple laser beacons, as opposed to multiple natural guide stars, are needed to achieve a useful degree of sky coverage. The use of laser beacons at multiple altitudes in a hybrid laser beacon configuration has been shown in previous papers to reduce both focus and tilt anisoplanatism. In this study we combine all three of these aspects. The hybrid beacon scenarios used in this study consists of multiple high altitude sodium beacons at 90 km and/or multiple low altitude Rayleigh beacons at 10 to 20 km. We present results for an 8-m class telescope for 2 and 3 different DM conjugate altitudes. For each of these MCAO configurations the following parameters are varied: number of Rayleigh beacons, number of Rayleigh beacon wavefront sensor (WFS) subapertures, Rayleigh beacon altitudes for the Rayleigh/sodium configuration, number of natural beacons for tip/tilt correction, and number of natural beacon WFS subapertures. When the WFS subaperture for the natural beacon is greater than 1 x 1 it contributes to the higher order correction in addition to being used for tip/tilt correction. Results are compared in terms of Strehl Ratio for the J, H, and K band.

Keywords: multi-conjugate adaptive optics, adaptive optics, multiple laser guide stars, laser guide stars, anisoplanatism

1. INTRODUCTION

MCAO has been suggested by several researchers[1–4] as a way of increasing the corrected FOV of a laser beacon adaptive optical (AO) system. MCAO means multiple DM's conjugate to multiple altitudes in the atmosphere. The DM's are typically conjugate to layers in the atmosphere where turbulence is predominant. As with a conventional AO system, the laser beacons are necessary in order to achieve a useful degree of sky coverage at high Strehl ratios. Natural beacons are still needed to measure tip/tilt due to the position uncertainty of laser beacons. Sky coverage is a function of such parameters as the observing scenario, the natural beacon density profile, atmospheric conditions and AO system parameters. It can be defined as the probability of achieving a Strehl ratio (SR) at least equal to some threshold based on the number of natural beacons that are sufficiently bright and close enough to the science field.[5] Based on the previous results,[6] the hybrid laser beacon AO configuration using the Rayleigh/sodium combination is the preferred method to start with since it has already been shown to reduce both focus and tilt anisoplanatism. Section 2 outlines the parameters of this study. Section 3 describes the natural beacon baseline, the best case, which is what the configurations in this study strive to emulate. Section 4 describes two different types of hybrid configurations, sodium/natural and Rayleigh/sodium, illustrates the results, and discusses the differences and the analysis of the runs. Section 5 discusses the runs, results and analysis for the Rayleigh/Rayleigh hybrid configuration. Section 6 summarizes this paper.

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2. PARAMETERS AND STUDY OUTLINE

This study is based on the 8 m class Gemini telescopes being built on Mauna Kea in Hawaii and on Cerro Pachon in Chili to observe astronomical objects in the infrared. The wavelengths used to estimate the AO performance are J (1.24 μrad), H (1.65 μrad), and K (2.2 μrad) bands. The laser beacon AO system under development for Gemini-South is fully compatible with MCAO. Gemini-North is built and had seen first light at the time of the study. The Altair natural beacon AO system will implement a single DM conjugate to 6.5 km in 2002. This paper uses the Gemini-North telescope model but adds additional DM's at peak locations of turbulence as seen in the C_n^2 profile shown in figure 1. The DM conjugate altitudes were chosen to be at 0, 4.5 and 9 km. The average seeing model was used for all cases with an r_o of 23.5 cm at a wavelength of 0.5 μm, θ_0 of 11.125 μrad, and f_g of 9.953 Hz. The effective aperture was 7.9 m with a linear central obscuration ratio of 0.152.

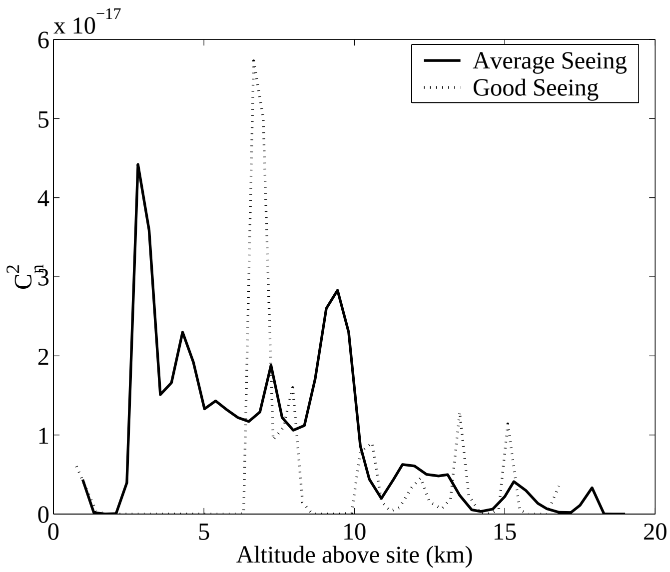


Figure 1. Representative Mauna Kea C_n^2 profiles.

The AO simulation code LACE was used to perform this study. LACE, named after a satellite, is based on an integrated analytical model for the principal first-order error sources in an AO system, namely fitting error, anisoplanatism, WFS measurements noise, and servo lag. See reference 7 for a description of this model.[7]

For this study additional DM's at 4.5 and 9.0 km were defined. The FOV for this configuration was set to 148.148 μrad with 12 x 12 subapertures at 0 km. The number of subapertures at 4.5 and 9 km were 14 x 14 and 16 x 16 respectively. However, due to less turbulence at the higher altitudes, the DM conjugate to 9 km can have twice the inter-actuator spacing resulting in 8 x 8 subapertures since a higher degree of correction at this altitude is not needed. This makes the number of actuators required at the DM's conjugate to 0, 4.5, and 9 km to be 13 x 13, 15 x 15, and 9 x 9 respectively. A small set of runs using 16 x 16 subapertures at 0 km, with the same DM conjugate altitudes of 0, 4.5, and 9.0 km and a corresponding FOV of 166.667 μrad were also run to illustrate the reduction in fitting error. These results will be addressed in section 4.2.

With an increase in FOV being investigated, the evaluation directions to encompass this FOV are included for proper evaluation. Performance over the FOV is a measure of how well the configurations correct for angular anisoplanatism. Nine evaluation directions were used on a square, the center, four edges and four corners of the field. A weighting was applied to each direction. The center had the highest weighting of 0.5, the edges were weighted with 0.1 each, and the corners were weighted with 0.025 each. For the 12 x 12 cases, the top right corner of the square field of view is at the location (148.148 μrad, 148.148 μrad). The rest follow the regular progression around the square field, with (0,0) at the center. The location of the beacons depends on how many were used, but were typically located at the center and corners of the square field. The FOV was defined from the center to an edge.

This study starts out with the case of five natural beacons and 2 or 3 DM's, resulting in the best case for these MCAO configurations. Since obtaining five natural beacons with the appropriate magnitude in the direction of interest will be next to impossible, the multiple laser beacon hybrid configurations are introduced to study which type of configuration will come closest to that achieved with 5 natural beacons. A form of the hybrid method using five sodium beacons with 12 x 12 subapertures and a combination of 1 to 5 natural beacons with varying subapertures of 1 x 1, 2 x 2, 4 x 4, and 6 x 6 was studied first. Natural beacon subapertures greater than one implies the beacons were of varying and reduced magnitudes compared to the sodium beacon and used for some higher order correction. Then 1 to 5 Rayleigh beacons were added to the 5 sodium beacons. Now the configuration consists of MCAO with multiple laser beacons at multiple altitudes, including a single natural beacon for tip/tilt correction. The Rayleigh beacons were of reduced power compared to the Sodium beacon and had varying subapertures of 2 x 2, 4 x 4 and 6 x 6. The altitude of the Rayleigh beacons was varied from 10 km, 15 km, and 20 km. A two DM configuration with DM's conjugate to 0 and 9 km, and 0 and 6.5 km was also studied. It was felt that a two DM configuration was worth looking at due to the reduced complexity. Both of these DM conjugate altitude configurations were studied due to the appropriate matching peaks in the C_n^2 profile and the fact that the Gemini-North telescope will implement a DM conjugate to 6.5 km. The study continues with only the use of Rayleigh beacons at multiple altitudes, again looking at the use of 2 and 3 DM's. 1 to 5 Rayleigh beacons with varying number of subapertures, 2 x 2, 4 x 4, 6 x 6, and 12 x 12, at altitudes of 10, 12, 15, and 18 km were studied for the lower altitude. For the higher altitude Rayleigh beacons at 20 km, 5 to 9 beacons were used at a constant 12 x 12 subapertures. The results for all of these configurations, natural beacon, sodium/natural, Rayleigh/sodium, Rayleigh/natural and Rayleigh/Rayleigh are reported in the form of SR's, for the center, edges, and corners of the square field. The SR's for the edges and corners gave an indication of how well the angular anisoplanatism or the field of view is being corrected.

3. NATURAL BEACON BASELINE

A baseline is needed by which to compare the multi-conjugate/multiple laser beacon configurations. This baseline consists of five natural beacons and 2 or 3 DM's. This is in effect the best case for each number of DM's that can be achieved using natural beacons. The sky coverage of suitable natural beacons with the appropriate magnitude in the direction of interest is most likely next to impossible. Therefore, it is necessary to study suitable substitutes that can get close to the five natural beacon results. The baseline is, therefore, five natural beacons, each with suitable magnitude to use 12 x 12 subapertures on the DM and WFS. To begin with, a varying number of subapertures were studied, from 1 x 1, 2 x 2, 4 x 4, 6 x 6, and 12 x 12. The smaller number of subapertures would model dimmer stars. The best case, however, would be the 12 x 12 subapertures.

Table 1 illustrates the SR results for the 12 x 12 subaperture case for the three different wavelengths of interest and for the center, edges and corners of the square field. This table includes the natural beacon SR's for a single DM conjugate to 0 km, two DM's conjugate to 0 km and 6.5 km, and three DM's conjugate to 0, 4.5, and 9 km. Notice the SR's in the edge and corner columns. The SR's dramatically increase with the number of DM's. This illustrates the reduction in angular anisoplanatism by the use of multiple DM's resulting in a larger corrected FOV. The value at the center of the field for the 1 DM case is lower than expected. This is due to the weighting in the evaluation directions. The full field for the 1 DM was presented even though this is not practical, just to show how the field of view cannot be corrected without MCAO. All three cases have identical on-axis SR's when only on-axis performance is weighted by the reconstruction algorithm. The values that are of use in this table are the data for the 2 and 3 DM's, and it is these results that will be attempted to be emulated using artificial laser beacons.

DM	**SR at 1.25 μm**			**SR at 1.65 μm**			**SR at 2.25 μm**		
	Center	Edge	Corner	Center	Edge	Corner	Center	Edge	Corner
1	0.486	0.038	0.021	0.651	0.112	0.056	0.775	0.261	0.147
2	0.651	0.354	0.244	0.782	0.55	0.44	0.87	0.713	0.627
3	0.735	0.526	0.508	0.839	0.691	0.677	0.906	0.812	0.803

Table 1. Five natural beacons with 1, 2, and 3 DM's, illustrates the reduction in angular anisoplanatism across the full field with increased number of DM's. The multiple DM cases provide a baseline for the hybrid cases. For each case the conjugate altitudes are: 1 DM conjugate to 0 km, 2 DM conjugate to 0 and 6.5 km, and 3 DM conjugate to 0, 4.5, and 9 km.

4. RAYLEIGH/SODIUM HYBRID BEACON CONFIGURATIONS

Now that a baseline has been established using the multiple natural beacons, the next step is to look at configurations using artificial beacons to determine if there is a suitable configuration that can achieve performance close to that of the baseline. As mentioned previously, the hybrid beacon type of configurations are most likely to give the desired results. Two of these configurations will be studied. First, the combination of multiple sodium beacons and one or more natural beacons. The natural beacons are used for both tip/tilt correction and some higher order correction depending on how many subapertures are used. If more than a single subaperture is used to collect the light from a natural beacon, then some higher order correction is accomplished. The second configuration is a combination of multiple sodium beacons, one or more Rayleigh beacons, and a single natural beacon for tip/tilt correction. The first configuration is studied to see what level of improvement can be gained with natural beacons. The use of multiple natural beacons of higher order magnitude would be difficult to achieve based on the sky coverage. The second configuration would be easier to achieve because of increased sky coverage for dimmer stars suitable for tip/tilt. Both configurations were investigated using 2 and 3 DM's.

4.1. Two and Three DM Runs and Results

This section studies both hybrid configurations using 2 and 3 DM's. For the 2 DM cases two different configurations with the DM's conjugate to 0 and 9 km, and 0 and 6.5 km were originally studied. The second case is the one chosen to report because the Gemini-North telescope is planning to implement a DM conjugate to 6.5 km in the year 2002. The 2 DM case is worth studying because of its reduced complexity compared to the 3 DM case. For the sodium/natural hybrid case, five sodium beacons were used with 12 x 12 subapertures along with 1 to 5 natural beacons. All of the natural beacons had a single subaperture (1 x 1) and were used for tip/tilt correction only. Figure 2 illustrates these results for both the 2 and 3 DM cases. The SR for the center, edge and corner of the field are shown. The SR improves with the increased number of beacons as expected. 10 % at the center, 30 % at the edges, and 37 % at the corners for beacons from 1 to 5 for the 2 DM case and 14 %, 33 %, and 42 % at the center, edge, and corner of the field respectively for the 3 DM case. The improvements from 1 to 2 beacons is more significant than from 2 to 5. These plots illustrate the increase in performance with the number of natural beacons. Results were obtained for all 3 wavelengths, 1.25 μm, 1.65 μm, and 2.25 μm, but the mid wavelength of 1.65 μm was chosen to report.

Figures 3 (a) and (b) show SR results for the 2 DM Rayleigh/sodium hybrid configuration which includes 5 sodium beacons with 12 x 12 subapertures, one to five Rayleigh beacons with varying subapertures, and one natural beacon with 1 x 1 subaperture. Figure 3 (a) illustrates the change in SR with the change in the number of Rayleigh beacon subapertures for three different altitudes; 10, 15, and 20 km for 5 Rayleigh beacons. The change between the number of subapertures 2 x 2, 4 x 4, and 6 x 6 is very small, 0.3 - 0.8 % at the center, 3 % at the edges and 0.5 - 1 % at the corners. The 6 x 6 subaperture case gives the best performance, however 4 x 4 subapertures is a reasonable choice to work with and is used as a basis for following runs. The difference in performance for 6 x 6 subapertures, based on the altitude of the Rayleigh beacons is also small. About 1 % at the center, 5 % at the edges and 7 % at the corners. The altitude performance for the 2 x 2 and 4 x 4 subaperture cases are similar. The higher altitude of 20 km produces the best overall performance. The altitude has more of an impact on the edge and corner results then on the center of the field. The zero case, also shown on this plot, needs to be mentioned because it represents only sodium beacons, and no Rayleigh beacons. This illustrates the improvement gained by the use of multiple beacons at multiple altitudes. There is a substantial improvement from the zero case to just 2 x 2 subapertures, 10 % at the center, 25 - 28 % at the edges, and 31 - 35 % at the corners. For each set of plots the center of the field is shown in the top set of traces, the edge of the field in the middle set of traces and the corner of the field in the bottom set of traces.

Figure 3 (b) illustrates the change in SR vs the number of Rayleigh beacons at 10, 15, and 20 km. Each Rayleigh beacon had 4 x 4 subapertures. Similar behavior in the results is seen here also. The change is very small with the increase in number of beacons from 1 to 5. It is 0.1 - 0.6 % at the center, and 3 % at the edges and corners. The difference in performance for 5 beacons based on altitude is 0.8 % at the center, 5 % at the edges, and 7 % at the corners. The results for 1 to 4 Rayleigh beacons are similar. When comparing the zero case to just 1 Rayleigh beacon, significant improvement is seen. 10 % at the center, 24 - 28 % at the edges and 30 - 35 % at the corners. Not much of a difference in SR's are observed for the 15 and 20 km cases for a low number (1 to 2) of Rayleigh beacons for all areas of the field. More of a difference is observed for 3 to 5 beacons, especially at the edges and corners, indicating a reduction in angular anisoplanatism and better correction over the field with increased number

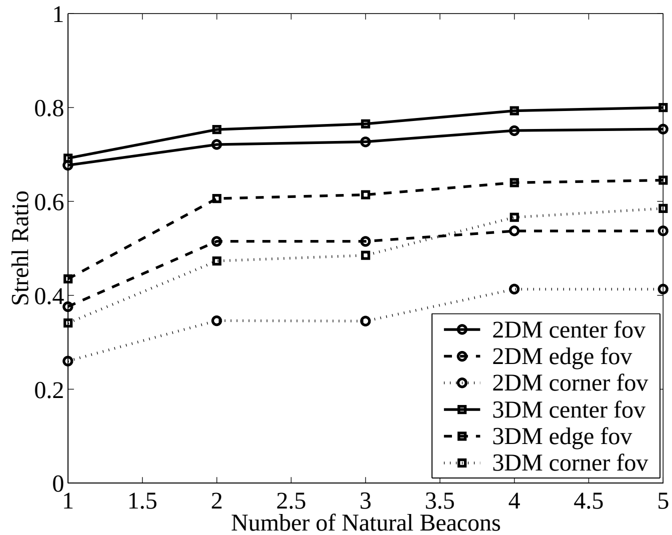


Figure 2. Strehl Ratio vs the number of natural beacons for the 2 and 3 DM sodium/natural hybrid configuration at the center, edge and corner of the field. The FOV is 148.148 μrad and the number of subapertures for the 5 sodium beacons and natural beacons are 12 x 12 and 1 x 1 respectively. Results at 1.65 μm are shown.

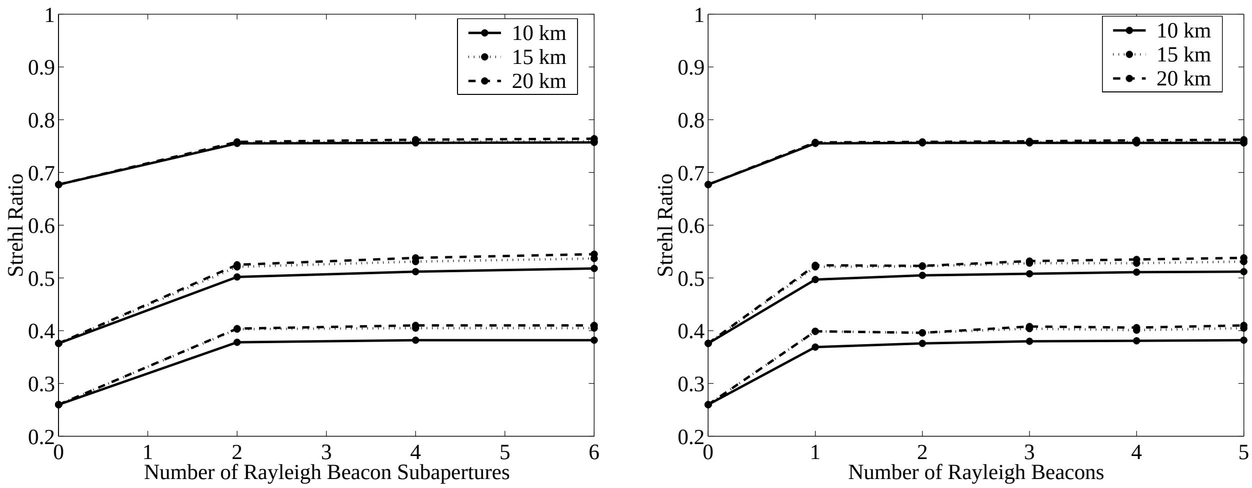
of beacons. This conclusion is difficult to discern from viewing the plots. The actual numbers were used for this conclusion. Based on the results from these two plots, an altitude of 20 km for the Rayleigh beacon is chosen for use in the final analysis.

Figure 4(a) and (b) illustrate the SR results from the 3 DM Rayleigh/sodium hybrid configuration. First notice that the SR's are better for the center, edge, and corner of the field than they were for the 2 DM case. Also notice that the improvement with altitude is more evident for the edges and corners of the field, 9 % and 14 % respectively. The improvement at the center of the field with altitude is small on the order of 1 %. Figure 4 (a) displays SR vs the number of Rayleigh beacon subapertures. The improvement from 2 x 2 to 6 x 6 subapertures is small as seen with the 2 DM case. 0.8 - 1 % at the center, 3 - 5 % at the edges, and 3 % at the corners. The 6 x 6 subaperture case is the best but not significantly. Therefore, 4 x 4 subapertures is a reasonable choice to make for future runs as seen with the 2 DM case. Choosing 4 x 4 subapertures over 6 x 6 reduces the laser power required to make the beacons. Improvement from the zero case, with no Rayleigh beacons, to 5 Rayleigh beacons with 2 x 2 subapertures is quite significant, 14 % at the center, 24 - 31 % at the edges, and 30 - 40 % at the corners.

Figure 4 (b) displays SR vs the number of Rayleigh beacons. The improvement from 1 to 5 Rayleigh beacons is small as seen with the 2 DM case results. 0.5 - 1.5 % at the center, 3 - 6 % at the edges, and 2 - 7 % at the corners. Improvement from the zero case to a single Rayleigh beacon is again quite significant; 13 % at the center, 25 - 29 % at the edges, and 31 - 37 % at the corners. As was seen with the 2 DM cases, the higher Rayleigh beacon altitude of 20 km gives the better performance.

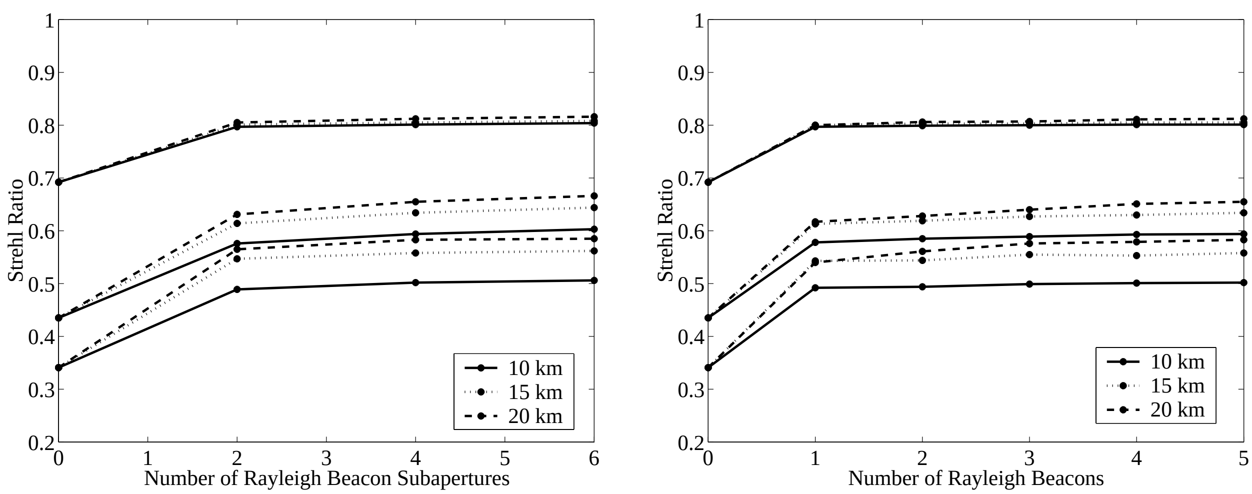
4.2. Analysis

From the results just presented, the 3 DM cases do give better performance over the 2 DM cases in all areas of the field. Starting with the sodium/natural configuration for both 2 and 3 DM's, the results shown in figure 2 are compared. The SR improvement from 2 to 3 DM's for the sodium/natural hybrid configuration with 1 natural beacon is 2 % at the center, 14 % at the edges, and 24 % at the corners. The improvement for 5 natural beacons is 6 % at the center, 17 % at the edges, and 29 % at the corners. The improvement is more dramatic at the edges and corners of the field, but more gradual for both cases after 2 to 3 beacons. Figures 3 and 4 present results for the Rayleigh/sodium hybrid configurations and they too illustrate the higher SR's for the 3 DM cases and, therefore, better performance in all areas of the field. The improvement for the 3 DM case over the 2 DM case for 5 Rayleigh beacons with 4 x 4 subapertures is 6 % at the center, 18 % at the edges and 30 % at the corners. This means that the increase in the number of DM's as well as the altitude, has an impact on performance of the system as seen in the improved SR's. One advantage of the 3 DM cases is more uniform performance across the field, which will



(a) (b)

Figure 3. Strehl Ratio results for the 2 DM Rayleigh/sodium hybrid configuration for the center, edge and corner of the field for 3 different Rayleigh beacon altitudes 10, 15, and 20 km. The FOV is 148.148 μrad, wavelength is 1.65 μm, and the number of subapertures for the 5 sodium beacons and natural beacon are 12 x 12 and 1 x 1 respectively. (a) Strehl ratio vs. the number of Rayleigh beacon subapertures for 5 Rayleigh beacons where the numbers on the x axis mean 1 x 1, 2 x 2, etc. (b) Strehl ratio vs. the number of Rayleigh beacons with 4 x 4 subapertures for each beacon.



(a) (b)

Figure 4. Strehl Ratio results for the 3 DM Rayleigh/sodium hybrid configurations for the center, edge and corner of the field at 3 different Rayleigh beacon altitudes 10, 15, and 20 km. The FOV is 148.148 μrad, wavelength is 1.65 μm, and the number of subapertures for the 5 sodium beacons and natural beacons are 12 x 12 and 1 x 1 respectively. (a) Strehl ratio vs the number of Rayleigh beacon subapertures for 5 Rayleigh beacons where the numbers on the x axis mean 1 x 1, 2 x 2, etc. (b) Strehl ratio vs the number of Rayleigh beacons for 4 x 4 subapertures.

make data reduction and image post processing simpler. So even with the increased complexity, there is substantial reason to go ahead and pursue the 3 DM hybrid configuration. Therefore, the 3 DM configuration is now discussed in greater depth.

Table 2 displays the data from both hybrid configurations and compares them to the best case configuration of 5 natural beacons with 12 x 12 subapertures shown on the top half of the table. The 16 x 16 data is displayed on the bottom half of the table. Data for all three wavelengths was generated but only 1.65 μm is reported in this paper. The single natural beacon cases are displayed at the beginning of each set of data to illustrate the need for multiple beacons to achieve correction over the full field. The SR's for the center, edge and corner of the field are 0.839, 0.691 and 0.677 respectively for the best case at 1.65 μm. A SR close to this value is desired using artificial beacons at two different altitudes for all parts in the field. For the sodium/natural hybrid beacon configuration with 5 sodium beacons and 1 natural beacon the SR in all parts of the FOV are lower indicating not as much reduction in angular anisoplanatism. By increasing the number of subapertures of the natural beacon to 6 x 6, therefore using the natural beacon for some higher order correction, the SR's at the center, edge and corner of the field increase to 0.818, 0.601 and 0.506 respectively, a very good improvement. The improvement at the edge and corner of the field, indicate better reduction in angular anisoplanatism. By increasing the natural beacons to 4, all with a single subaperture, meaning that the natural beacons are only used for tip/tilt correction, the SR's at the center, edge and corner are 0.793, 0.64 and 0.566 respectively where the SR's at the edges and corners have increased over the previous case. This indicates that the increase in the number of natural beacons plays a role in reducing the angular anisoplanatism and increasing the corrected FOV. Either of these two configurations will be difficult to achieve at all times due to sky coverage.

Natural		**Sodium**		**Rayleigh**			**SR at 1.65 μm**		
N_n	S_n	N_s	S_s	N_r	$A_r(km)$	S_r	Center	Edge	Corner
1	12						0.347	0.031	0.012
5	12						0.839	0.691	0.677
1	1	5	12				0.692	0.435	0.341
1	6	5	12				0.818	0.601	0.506
4	1	5	12				0.793	0.64	0.566
1	1	5	12	5	20	4	0.812	0.655	0.583
1	16						0.442	0.52	0.018
5	16						0.889	0.705	0.671
1	1	5	16				0.723	0.419	0.315
1	6	5	16				0.857	0.628	0.537
4	1	5	16				0.834	0.636	0.566
1	1	5	16	5	20	4	0.855	0.69	0.614

Table 2. This table represents a summary of the data for three DM's at conjugate altitudes of 0, 4.5, and 9 km with a FOV of 148.148 μrad. The results for the 12 x 12 subapertures are on the top half of the table and the results for the 16 x 16 subapertures are at the bottom half of the table. N_n, N_s, and N_r represent the number of natural, sodium and Rayleigh beacons respectively. The column labeled S with the appropriate subscript is the number of subapertures used to collect the light for each type of beacon. A_r under Rayleigh is the altitude of the Rayleigh beacon in km.

Rayleigh beacons are now added, resulting in 5 sodium beacons, 5 Rayleigh beacons and 1 natural beacon for tip/tilt correction. The sodium beacons have 12 x 12 subapertures as before and the Rayleigh beacons have 4 x 4 subapertures, based on the previous sets of data illustrating the increase in subapertures did not improve the overall SR significantly. The altitude of the Rayleigh beacons is chosen at 20 km based on previous results. It is also the highest reasonable altitude due to the amount of particulate in the atmosphere available to scatter light. The natural beacon was only used for tip/tilt correction, and therefore, used only a single subaperture (1 x 1). The SR's at 1.65 μm at the center, edge, and corner of the field are 0.812, 0.655 and 0.583 respectively. This is very close, about 0.7 %, to the sodium/natural beacon case with 6 x 6 subapertures and only about 3 % lower than the best case configuration of 5 natural beacons. At the edges and corners, this case is 8 % and 6 % better then the 6 x 6 subaperture case, and 5 % and 14 % lower then the best case. This configuration does not correct as well as the best case at the edges and corners of the field, but does do better than all of the other configurations.

To look at further improvements, 16 x 16 subaperture cases were run to show the improvement due to fitting error. Fitting error occurs due to the non-zero size of the subapertures. Two different types of runs were executed. First, to keep the parameters constant including the FOV of 148.148 μrad, the number of subapertures were adjusted at each altitude, 0, 4.5 and 9 km to 16 x 16, 18 x 18 and 10 x 10 subapertures respectively. To make a better comparison to the original 12 x 12 subaperture set of data shown on the top half of table 2, it was necessary to keep as many of the parameters constant. By comparing this set of data to the bottom half of table 2, an improvement is seen on the order of about 5 % due to fitting error.

It was also decided to run hybrid beacon configurations with 16 x 16 subapertures, the same DM conjugate altitudes of 0, 4.5, and 9 km and a FOV of 166.667 μrad. The number of subapertures at the two higher altitudes were increased to 19 x 19 and 11 x 11. This data is shown in table 3. There is improvement at the center of the field but not at the edges and corners of the field for the Rayleigh/sodium hybrid beacon configuration compared to the same results in table 2. This is due to the increase in the FOV. So, it can be concluded from these two sets of 16 x 16 sets of data that the increase in the number of subapertures does reduce the fitting error, but with increased FOV does not reduce the angular anisoplanatism as much as is seen in the smaller FOV. This also illustrates that larger FOV's are harder to correct as in conventional optics.

Natural		**Sodium**		**Rayleigh**			**SR at 1.65 μm**		
N_n	S_n	N_s	S_s	N_r	$A_r(km)$	S_r	Center	Edge	Corner
1	16						0.399	0.04	0.015
5	16						0.901	0.716	0.719
1	1	5	16				0.733	0.286	0.214
1	6	5	16				0.873	0.604	0.499
4	1	5	16				0.84	0.623	0.539
1	1	5	16	5	20	4	0.861	0.639	0.543

Table 3. This table represents a summary of the data for three DM's at conjugate altitudes of 0, 4.5, and 9 km, with a FOV of 166.667 μrad and have a maximum of 16 x 16 subapertures. N_n, N_s, and N_r represent the number of natural, sodium and Rayleigh beacons respectively. The column labeled S with the appropriate subscript is the number of subapertures used to collect the light for each type of beacon. A_r under Rayleigh is the altitude of the Rayleigh beacon in km.

5. RAYLEIGH/RAYLEIGH HYBRID BEACON CONFIGURATIONS

In a continuation of the study of MCAO with hybrid or multiple altitude laser beacons, MCAO with only Rayleigh beacons is studied as a means to increase the telescope field of view and improve performance over single DM and single Rayleigh beacons without the use of sodium beacons. This type of system has some interest in the astronomical community[8,9] and is studied as a possibility since the technology for sodium beacons, especially for correction in the visible wavelengths, remains difficult. The parameters are identical to that in the previous sections. Both 2 DM and 3 DM cases are studied. For the 2 DM case, two configurations, with the DM's conjugate to 0 and 6.5 km and to 0 and 9 km are studied. The first configuration results are reported in this paper. Altitudes of 10 and 20 km were used with 5 Rayleigh beacons each to start with. Variations on this were also run, including differing number of beacons, different lower beacon altitudes, and single and multiple natural beacons for the tip/tilt correction. The natural beacons had single subapertures for tip/tilt corrections in addition to multiple subapertures for additional higher order correction. Performance was evaluated at the same FOV as before, 148.148 μrad and the same wavelengths in the J, H, and K bands.

5.1. Two and Three DM Runs and Results

The initial results were somewhat promising for a Rayleigh/Rayleigh hybrid configuration. The results are not as good as the Rayleigh/sodium hybrid configuration described in section 4, but encouraging none-the-less for this type

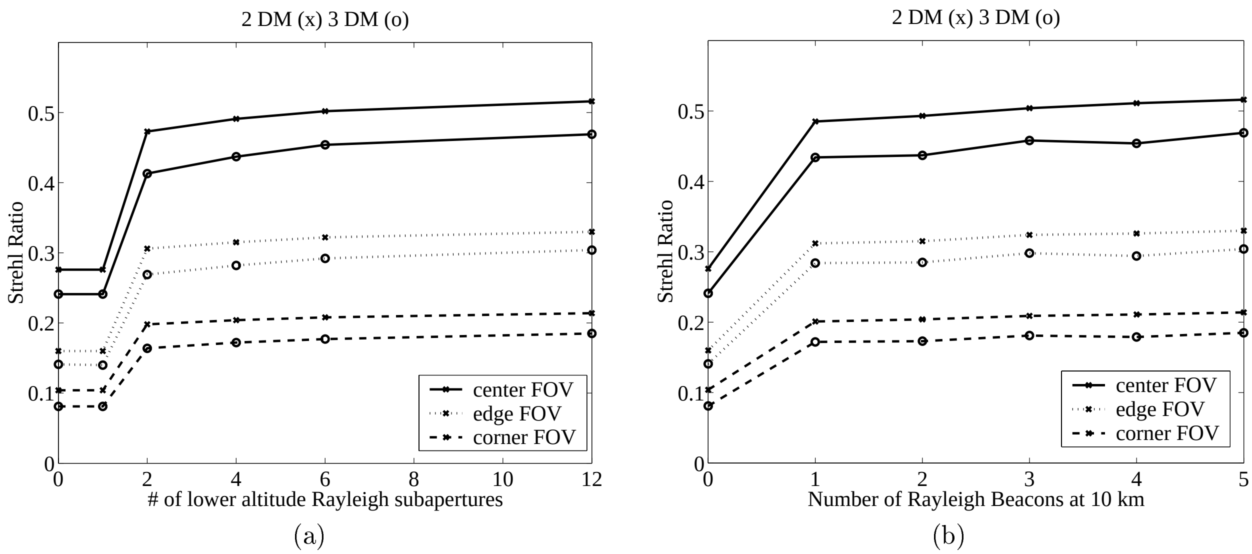


Figure 5. Strehl Ratio for the 2 DM and 3 DM Rayleigh/Rayleigh hybrid configurations for the center, edge and corner of the field. The FOV is 148.148 μrad, wavelength is 1.65 μm, and the number of subapertures for higher altitude Rayleigh beacons is 12 x 12 for (a), 12 x 12 for all Rayleigh beacons in (b), and 1 x 1 for all the natural beacons. (a) Strehl ratio vs the number of lower altitude Rayleigh beacon subapertures where the numbers on the x axis mean 2 x 2, 4 x 4, etc. (b) Strehl ratio vs the number of low altitude (10 km) Rayleigh beacons.

of configuration. Initially the 2 DM with the DM's conjugate to 0 and 6.5 km produced the best results. It is believed that this is because the DM conjugate altitude at 9 km is close to the lower beacon layer of 10 km, resulting in many unobservable actuators. The nearest nearest-neighbor "slaving" routines used in LACE to control unobservable actuators are evidently not adequate for this case. To try to improve the results the number of subapertures at the high altitude DM were changed from 8 x 8 to 4 x 4 (actuators from 9 x 9 to 5 x 5) which changed the inter-actuator spacing from D/6 (1.333 m) to D/3 (2.666 m). Where D is the diameter of the telescope and equal to 8 m. This improved the performance for most of the 3 DM cases but not much for the 2 DM cases. There was virtually no improvement for the 3 DM cases which had multiple natural beacons. Negative improvement was noticed on the edges and corners for the 2 DM cases, possibly resulting in the reduced number of subapertures at the high altitude DM. The 2 DM cases with DM's conjugate at 0 and 6.5 km still showed better results when the best cases of each is compared.

Figure 5 illustrate several different comparisons to look for any combinations that stand out as possible configurations that would be useful in a real system. Figure 5 (a) shows the change in SR as a function of the number of subapertures in the lower altitude Rayleigh beacon. For these cases the number of subapertures at the higher altitude Rayleigh beacon at 20 km is held at 12 x 12 while the number of subapertures on the lower altitude Rayleigh beacon is varied from the zero case to 1 x 1, 2 x 2, 4 x 4, 6 x 6, and 12 x 12 subapertures. The zero case is only for the high altitude Rayleigh beacons. The altitude of the lower Rayleigh beacons are held at 10 km. From this figure it is evident that from 2 x 2 to 6 x 6 subapertures the increase in performance is small and is relatively flat from 6 x 6 to 12 x 12 subapertures. The increase in performance from 2 x 2 to 12 x 12 subapertures is 8 % at the center, 7 % at the edges and corners of the field for the 2 DM and 12 % at the center and edges, and 11 % at the corners for the 3 DM case. Similar behavior is seen for both cases with better performance demonstrated for the 2 DM case. This would indicate that a small number of subapertures and, therefore, a lower power laser would be possible for the lower altitude Rayleigh beacons and still obtain reasonable performance. The zero case and the single subaperture case are virtually identical. The improvement from the zero case to 2 x 2 subapertures is 42 % for the center, 48 % for the edges, and 47 % for the corners. The improvement seen from the zero case for the 3 DM is identical at the center and edges and 51 % at the corners. The 2 DM case with DM's conjugate to 0 and 6.5 km is the top trace for each set and the 3 DM case the bottom trace. The center, edge, and corner of the field are represented in the top, middle, and bottom set of traces on each plot.

Figure 5 (b) is a plot of SR vs the number of Rayleigh beacons at the lower altitude of 10 km. For this case, all the subapertures of all the beacons are held at 12 x 12, and the number of the high altitude Rayleigh beacons are held at 5 at an altitude of 20 km. There is a sharp increase from the zero case to a single Rayleigh beacon, but after that the increase is very gradual and not very significant. The improvement from the zero case to a single low altitude Rayleigh beacon is 43 % at the center, 48 % at the edges and corners for the 2 DM case. The improvement is similar for the 3 DM case. The improvement from 1 to 5 low altitude Rayleigh beacons is about 6 % over the whole field for the 2 DM case and 7 % for the 3 DM case. Similar behavior is shown for both the 2 DM and 3 DM configurations. Both figures 5 (a) and (b) show an improvement over 5 Rayleigh beacons at a single altitude, in all parts of the field, but the improvement is not that significant as both the number of subapertures and number of Rayleigh beacon are increased.

To look at additional possibilities for increased performance, the altitude of the lower Rayleigh beacons are increased and the number of higher altitude Rayleigh beacons are increased from 5 to 9. When the lower altitude Rayleigh beacon is increased from 15 to 18 km the 3 DM case shows equivalent or better performance. This is the only occurrence of better performance for the 3 DM case. The performance for the 2 DM case is relatively flat. The change in the number of subapertures for the lower altitude Rayleigh beacon was not run for the 15 or 18 km case. If they were, they may show slightly different results due to the decrease in the number of particulate at the higher altitude. However, the trends would be the same. More subapertures and/or higher laser powers may be required for these higher altitude, lower Rayleigh beacons. For these cases, all of the Rayleigh beacon subapertures are kept at 12 x 12. When the higher altitude Rayleigh beacons are increased from 5 to 9 the increase is very gradual and similar for both 2 and 3 DM cases. The altitude and number of subapertures remains the same at 20 km and 12 x 12. The number of lower altitude Rayleigh beacons remain at 5 with 6 x 6 subapertures at an altitude of 10 km. A few cases where the number of lower altitude Rayleigh beacons were increased was looked at briefly, but the improvement was not as significant.

It should be noted that the data was generated with the inter-actuator spacing on the high altitude DM of D/6 for the 2 DM cases and D/3 for the 3 DM cases. This produced the best results for each case. Also, as in the Rayleigh/sodium hybrid results discussed in the first part of this paper, the FOV of the 2 DM and 3 DM cases were kept at 148.148 μrad, with the DM's conjugate to 0 and 6.5 km for the 2 DM cases and 0, 4.5, and 9 km for the 3 DM cases.

5.2. Analysis

After critiquing the data just discussed, the "best" configuration from each of the type was chosen and combined in a few select runs. These results are included in table 4. Taking a closer look at the data, illustrates the improvement of a Rayleigh/Rayleigh hybrid beacon configuration for both the 2 and 3 DM cases at 1.65 μm. Line 1 shows a single natural beacon with 12 x 12 subapertures which does not correct well across the field of view, illustrating the need for multiple beacons. Line 2 shows 5 natural beacons with 12 x 12 subapertures. This is the best case, exhibiting high SR's at the center, edge and corner of the field. It is these results that the different configurations will try to emulate. A note here on the best case: the results differ slightly from those shown in table 1. This is due to increased numerical accuracy for one of the inputs for the 2 DM case, and to the D/3 inter-actuator spacing on the high altitude DM for the 3 DM case. A single Rayleigh beacon at 20 km, lines 3 and 4, does not correct well across the field. The configuration in line 4 does better due to the multiple subapertures and higher order correction from the natural beacon. Lines 5 and 6 add multiple Rayleigh beacons at 20 km. Line 5, with a single subaperture for the natural beacon is much better then line 3, but far from the best case. By increasing the number of subapertures on the natural beacon to 6 x 6, the performance is improved by about 60% across the field for the 2 DM case and 65% to 72% for the 3 DM case. Lines 7 to 10 illustrate Rayleigh beacons at two altitudes with 6 x 6 and 12 x 12 subapertures for the lower altitude Rayleigh beacons and 1 x 1, 2 x 2, and 6 x 6 for the natural beacon. The improvement from line 7 to line 9 by just increasing the natural beacon subaperture from 1 x 1 to 2 x 2 is about 18% to 28% for the 2 DM and about 28% to 32% for the 3 DM case across the full field of view. This substantial increase in performance for 1 subaperture difference in the natural beacon might be worth looking into regarding sky coverage issues. The increase in performance gained with the use of 6 x 6 subapertures for the natural beacon would not be practical due to the sky coverage issues. It is also noted that when the number of subapertures increase for the natural beacon, the 3 DM configurations do better. To further improve on the configuration on line 7 without having to consider a brighter natural beacon, the number of beacons at the higher altitude was increased and the altitude of the lower altitude beacons along with the number of subapertures for the lower altitude Rayleigh beacons was optimized. Lines

11 to 15 show these results. Notice that line 14 for both 2 DM and 3 DM have similar performance, with the 3 DM have slightly better performance at the edge and corners.

Therefore, based on figure 5 and the actual numerical values shown in table 4, it would be reasonable to say that a 2 DM system with 7 beacons at 20 km, 5 beacons with 6 x 6 subapertures or as few as 1 beacon with 12 x 12 subapertures at 15 km, and 1 tip/tilt natural beacon could be used to produce a Rayleigh/Rayleigh hybrid system with performance that exceeds the performance of a single or multiple Rayleigh beacons at a single altitude across the full field. That is without using multiple subapertures for the natural beacon. If the multiple natural beacon subapertures were a possibility or a 3 DM configuration is preferred, then the configuration with 5 beacons at 20 km, 5 beacons at 10 km with 12 x 12 subapertures, and 1 natural beacon with 2 x 2 subapertures (line 9) would be an option.

Natural		**Rayleigh**						**2 DM: SR at 1.65 μm**			**3 DM: SR at 1.65 μm**		
N_n	S_n	N_r	A_r	S_r	N_r	A_r	S_r	Center	Edge	Corner	Center	Edge	Corner
1	12							0.653	0.09	0.042	0.382	0.043	0.014
5	12							0.779	0.547	0.438	0.815	0.636	0.544
1	1	1	20	12				0.018	0.015	0.013	0.018	0.015	0.013
1	12	1	20	12				0.639	0.273	0.149	0.657	0.242	0.108
1	1	5	20	12				0.276	0.16	0.104	0.241	0.141	0.081
1	6	5	20	12				0.668	0.418	0.26	0.702	0.467	0.287
1	1	5	20	12	5	10	12	0.516	0.33	0.214	0.469	0.304	0.185
1	1	5	20	12	5	10	6	0.502	0.322	0.208	0.454	0.292	0.177
1	2	5	20	12	5	10	12	0.632	0.417	0.263	0.642	0.447	0.269
1	6	5	20	12	5	10	12	0.696	0.442	0.274	0.728	0.496	0.308
1	1	7	20	12	1	15	2	0.529	0.325	0.208	0.492	0.316	0.19
1	1	7	20	12	5	15	2	0.554	0.347	0.223	0.547	0.355	0.216
1	1	7	20	12	1	15	6	0.536	0.331	0.212	0.5	0.321	0.194
1	1	7	20	12	5	15	6	0.57	0.357	0.23	0.57	0.376	0.234
1	1	7	20	12	1	15	12	0.54	0.334	0.214	0.508	0.327	0.197

Table 4. This table represents a summary of the data for 2 and 3 DM's at conjugate altitudes of 0 and 6.5 km and 0, 4.5, and 6.5 km respectively. The FOV is 148.148 μrad with a maximum of 12 x 12 subapertures. N_n and N_r represent the number of natural and Rayleigh beacons respectively. The column labeled S with the appropriate subscript is the number of subapertures used to collect the light for each type of beacon. A_r is the altitude of the Rayleigh beacons in km.

6. CONCLUSIONS

This paper covered a wide variety of MCAO/multiple laser beacon configurations for an 8 m class telescope. MCAO configurations included both 2 DM, due to simpler implementation for Rayleigh/sodium and better performance for Rayleigh/Rayleigh, and 3 DM which proved to have better performance for the Rayleigh/sodium hybrid configuration. The number of beacons and altitudes of the beacons were widely studied. The combinations included sodium/natural, Rayleigh/natural, Rayleigh/sodium and Rayleigh/Rayleigh. Varying the number of lower altitude laser beacons was studied, while a higher number always appeared optimum for the higher altitude beacon. Differing the number of subapertures for the lower altitude beacon was studied along with differing the number of subapertures for the natural beacon. The natural beacon is needed for tip/tilt correction. However, increasing the number of subapertures allowed it to also be used for some higher order correction. The Rayleigh/natural and sodium/natural runs were used for comparison as to what types of results could be achieved with these configurations. The best configuration for each hybrid beacon system resulting from this study are summarized in table 5. The Rayleigh/sodium with 3 DM's, 5 sodium beacons with 12 x 12 subapertures, 5 Rayleigh beacons with 4 x 4 subapertures at 20 km, and 1 tip/tilt natural beacon is the best for this configuration. The Rayleigh/Rayleigh with 2 DM's, 7 Rayleigh beacons at 20 km with 12 x 12 subapertures, 5 Rayleigh beacons at 15 km with 6 x 6 subapertures, and 1 tip/tilt natural beacon is the best for this configuration. If the 3 DM case is considered for the Rayleigh/Rayleigh configuration, the most reasonable would be either the one mentioned above for the 2 DM due to similar results, or the 5 Rayleigh beacons at 20 km with 12 x 12 subapertures, 5 Rayleigh beacons at 10 km with 12 x 12 subapertures, and 1 tip/tilt natural beacon with 2 x 2 subapertures. Comparing these "best" configurations, the 3 DM Rayleigh/sodium is far superior than the 2 or 3 DM Rayleigh/Rayleigh configurations.

DM	**Natural**		**Sodium**		**Rayleigh**						**SR at 1.65 μm**		
	N_n	S_n	N_s	S_s	N_r	A_r	S_r	N_r	A_r	S_r	Center	Edge	Corner
3	1	1	5	12							0.692	0.435	0.341
3	1	1	5	12	5	20	4				0.812	0.655	0.583
2	1	1			5	20	12				0.276	0.16	0.104
2	1	1			7	20	12	5	15	6	0.57	0.357	0.23
3	1	2			5	20	12	5	10	12	0.642	0.447	0.269

Table 5. This table represents a summary of the "best" configurations for the sodium/natural, Rayleigh/natural, Rayleigh/sodium and Rayleigh/Rayleigh. The FOV is 148.148 μrad with a maximum of 12 x 12 subapertures. DM represent the number of DM's for the configuration. The columns labeled N with the appropriate subscript represent the number of natural, sodium, and Rayleigh beacons. The columns labeled S with the appropriate subscript are the number of subapertures used to collect the light for each type of beacon. A_r is the altitude of the Rayleigh beacons in km.

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